Stability of pancake vortices in stratified-rotating fluids

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In stably stratified rotating fluids, vortices have a pancake shape with a small thickness compared to their radial extent. An example is Mediterranean eddies (Meddies) which are formed by salty water from Mediterranean sea [1] and found in the Atlantic ocean. To understand the stability of such vortices, the linear stability of an isolated axisymmetric pancake vortex is studied in stratified-rotating fluids. The base vortex is chosen to have a Gaussian angular velocity profile in radial (r) and vertical (z) direction as

$$\Omega = \Omega_0 \exp\left(-\frac{r^2}{R_0^2} - \frac{z^2}{Z_0^2}\right). \tag{1}$$

Here, Ω_0 is maximum angular velocity, Z_0 the half thickness and R_0 the radius. The base density inside the vortex is deduced from the thermal-wind relation. The stability problem is solved numerically by discretizing the linearized Boussinesq equations with finite element methods and by using the Krylov-Schur method of SLEPc library for the eigenvalue problem [2]. We investigate the most unstable mode as functions of azimuthal wavenumber m, Froude number $F_h = \Omega_0/N$, Rossby number $Ro = \Omega_0/\Omega_b$, Reynolds number Re and aspect ratio $\alpha = Z_0/R_0$ where N the Brunt-Väisälä frequency and Ω_b the ambient rotation. The results show that the pancake vortex can be unstable to many kinds of instabilities depending on the Froude and Rossby numbers: centrifugal instability (Fig. 1a) shear instability (Fig. 1b), gravitational instability ([3]), bending instability ([4]) and baroclinic instability (Fig. 1c). Their origins are identified thanks to instability criteria or by their resemblence to the 2D or 3D instabilities in the columnar limit ($\alpha \to \infty$). We will present their domains of existence in the parameter space and their mechanisms of selection depending on the confinement due to the pancake shape and the Reynolds number. The results will be compared to previous stability studies ([3, 5, 6, 7]), experiments ([8]) and observations in the ocean ([1]).

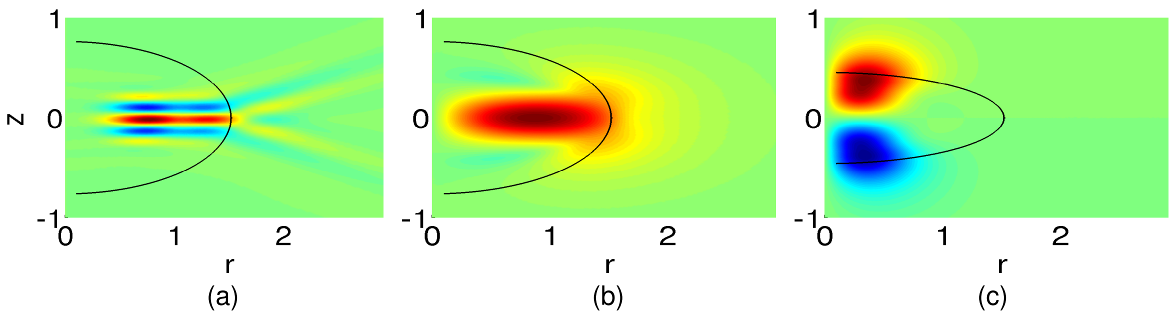

Fig. 1: Radial velocities of the most unstable mode for different control parameters: Centrifugal instability for $\alpha = 0.5, F_h = 0.5, Ro = -50, m = 2,$ and $Re = 10000$ (a), shear instability for $\alpha = 0.5, F_h = 0.05, Ro = 0.1, m = 2,$ and $Re = 10000$ (b) baroclinic instability for $\alpha = 0.3, F_h = 0.1, Ro = 0.2, m = 2,$ and $Re = 10000$ (c). The line shows the contour of the basic vortex where $\Omega = 0.1\Omega_0$.

References

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