Parametric subharmonic instability and energy cascades from internal modes in non-uniformly stratified fluid

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Introduction

Waves can become unstable through parametric subharmonic instability (PSI) by transferring energy irreversibly to a pair of waves with collectively resonant wavenumber and frequency [1, 2]. It is well established that two-dimensional internal plane waves and modes in uniformly stratified fluid efficiently transfer energy to smaller scale waves and ultimately turbulent mixing through PSI [3]. Recently it has been shown that PSI acts not just upon plane internal waves but also upon internal wavepackets [4] and internal wave beams [5, 6, 7, 8]. All of these studies considered internal waves in uniformly stratified fluid.

The numerical simulations of MacKinnon & Winters [9] predicted PSI should act efficiently to disrupt the internal tide. However, while in situ observations showed the presence of PSI, it was not found to be appreciable [10, 11]. One reason for the discrepancy between simulations and observations is that the former examined an internal mode in uniformly stratified fluid whereas, in reality, the internal tide is manifest as the sinusoidal oscillation of the thermocline; it is an interfacial wave.

The resonant condition of PSI for a one-dimensional interfacial wave in a two layer fluid is so restrictive that the instability is suppressed. However, at a thick interface like the thermocline, PSI may occur by exciting subharmonic waves with vertical as well as horizontal structure. Certainly in the limit of uniform stratification, the occurrence of PSI is robust.

The question remains, how does the onset and growth of PSI depend upon the interface thickness?

Methodology

Through theory and numerical simulations, we investigate the efficiency of PSI in extracting energy from interfacial waves as it depends upon the thickness of the interface.

We consider a two-dimensional Boussinesq fluid in a horizontally periodic domain of uniform depth. The background is stationary and the density profile is taken to be piecewise-linear so that one can write explicit analytic solutions for the vertical structure of small-amplitude internal modes. From this basic state, standard perturbation theory is employed to determine the reduced set of ordinary differential equations describing the coupling between the parent and offspring modes. Thus the stability and growth rate of PSI, if it occurs, is assessed and compared with the results of fully nonlinear numerical simulations.

Beyond examining the onset of instability, the numerical simulations are run for long times in order to examine the rate of energy transfer from a low mode internal wave to small scale disturbances as it depends upon the thickness of the interface and the modal structure of the parent wave.

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