Optimal mixing of buoyant jets and plumes in stratified fluids: theory and experiments.

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Abstract

We present results from an experimental and theoretical study of the influence of ambient fluid stratification on buoyant miscible jets and plumes. Given a fixed set of jet/plume parameters, and an ambient fluid stratification sandwiched between top and bottom homogenous densities, a theoretical criterion is identified showing how step-like density profiles constitute the most effective mixers within a broad class of stable density transitions. This is assessed both analytically and experimentally, respectively by establishing rigorous a priori estimates on generalized Morton-Taylor-Turner (MTT) models, and by studying a critical phenomenon determined by the distance between the jet/plume release height with respect to the depth of the ambient density transition. For fluid released sufficiently close to the background density transition, the buoyant jet fluid escapes and rises indefinitely. For fluid released at locations lower than a critical depth, the buoyant fluid stops rising and is trapped indefinitely. We develop a mathematical formulation providing rigorous estimates on MTT models, by establishing nonlinear jump conditions and an exact critical-depth formula in good quantitative agreement with the experiments. Our mathematical analysis provides rigorous justification for the critical trapping/escaping criteria, first presented in Caulfied and Woods (1998), within a class of algebraic density decay rates. Further, the analysis uncovers surprising differences between the Gaussian and Top-hat profile turbulent entrainment closures concerning initial mixing of the jet and ambient fluid. Laboratory experimental results and comparisons with the theory will be discussed.

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