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Dimensional transition of energy cascades in stably stratified thin fluid layers

A. Sozza[1]*, G. Boffetta[1], P. Muratore-Ginanneschi[2], S. Musacchio[3]

[1] Department of Physics and INFN, University of Torino, via P.Giuria 1, 10125 Torino, Italy
[2] Department of Mathematics and Statistics, University of Helsinki PL 68, FIN-00014 Helsinki, Finland
[3] Universite de Nice Sophia Antipolis, CNRS, LJAD, UMR 7351, 06100 Nice, France

Introduction

The turbulent dynamics of flows confined in quasi two-dimensional geometries exhibits a rich and interesting phenomenology. In such situation, a mixture of 2D and 3D dynamics can emerge [1]. In particular, when the thickness L_z of the fluid layer is smaller than the length scale of the forcing L_f, the turbulent cascade of kinetic energy splits in two parts. A fraction of the energy is transferred toward small, viscous scales as in 3D turbulence, while the remnant energy undergoes an inverse cascade toward large scales as in 2D turbulence. The key parameter which determines the energy flux of the two cascades is the ratio $S = L_z/L_f$ between the confining scale and the length scale of the forcing [2]. Stratified flows play a relevant role in geophysical systems, and clearly affects the effective dimensionality of turbulence.

DNS of Boussinesq equations

The dynamics of an incompressible flow, stably stratified in the vertical direction by a mean density gradient $\rho = \rho_0 - \gamma(z - \theta)$, is governed by the Boussinesq equations [3],[4]:

$$\begin{aligned} \partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} &= -\nabla p - N^2 \theta \hat{e}_3 + \nu \nabla^2 \boldsymbol{u} \\ \partial_t \theta + \boldsymbol{u} \cdot \nabla \theta &= w + \kappa \nabla^2 \theta \end{aligned} \qquad (1)$$

We investigate the effect of stratification on turbulent flows in quasi-two-dimensional geometry, by means of a set of high-resolution numerical simulations, focusing on the mechanisms of transfer of kinetic and potential energy. We will present a detailed analysis of the statistics of the energy dissipation rates and energy exchange rates and of the spectral fluxes of potential and kinetic energy. We also discuss the role of vortex stretching and enstrophy flux in the transfer of kinetic energy into potential energy at small scales.

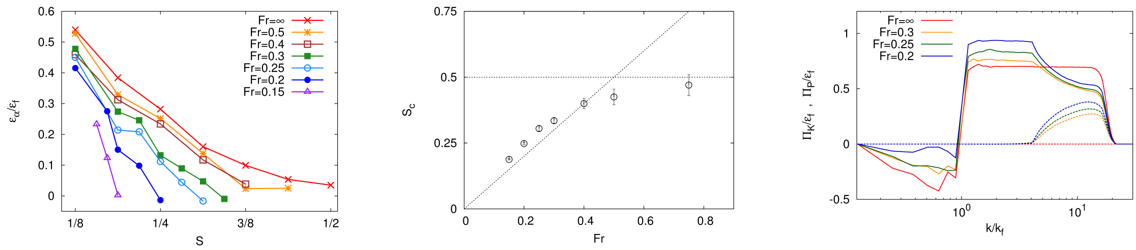

Fig. 1: (a) Growth rates of kinetic energy ε_α normalized with the energy input ε_f, as a function of the aspect ratio $S = L_z/L_f$ for different Fr. (b) Critical aspect ratio S_c as a function of Fr. (c) Spectral fluxes of kinetic energy (solid lines) and potential energy (dotted lines) as a function of the normalized wavenumber k/k_f, at fixed aspect ratio $S = 1/4$ and Froude number $Fr = 0.2$, $Fr = 0.25$, $Fr = 0.3$, $Fr = \infty$ from top to bottom.

We show that, when the vertical scale becomes comparable with the buoyancy scale, the growth of kinetic energy is gradually suppressed and a joint forward cascade of kinetic and potential energy develops. The critical aspect ratio, for which energy growth is totally arrested, scales linearly with Froude and corresponds to the typical length scale of the forming layers. To explain this behavior we argue that the density scalar field could represent a new channel for the dissipation of energy. In conclusion, stable stratification clearly affect the dimensionality of turbulence, as it inhibits inverse energy cascade and favors three-dimensional features.

References

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*corresponding author: sozza@infn.to.it