

Large eddy simulations of stratified turbulence at the buoyancy scale

Sina Khani* and Michael L. Waite

University of Waterloo, 200 University Avenue West, Waterloo, ON, Canada N2L 3G1

Introduction

Direct numerical simulation of stratified turbulence is very challenging in term of computational costs because it requires resolution of all scales from large energy containing scales down to the Ozmidov scale and down further to the Kolmogorov scale. Large eddy simulations (LES) is an alternative approach that resolves only large scales and parametrize the subgrid scale (SGS) motions. In this talk, the effects of resolving the buoyancy scale L_b in LES of stratified turbulence, when the grid spacing Δ is around the Ozmidov scale L_o or smaller, are discussed.

Materials and Methods

Common SGS models, the Kraichnan (1976) and Smagorinsky (1963) models, along with the dynamic Smagorinsky model (e.g. Germano et al., 1991), are considered. We investigate the maximum filter scale Δ for which the fundamental dynamics of stratified turbulence, including a downscale energy cascade in the horizontal direction and breakdown of layers into the Kelvin-Helmholtz (KH) instabilities, are captured. We consider strongly stratified turbulence, i.e. with

$$Fr_\ell = \frac{u}{N\ell} \ll 1, \quad (1)$$

in which quasi-horizontal vortical motions dominate large-scale dynamics. Here, Fr_ℓ is the Froude number; and u , N and ℓ are characteristic velocity scale, buoyancy frequency and length scale, respectively. Vortically forced stratified turbulence, where the forcing term is applied to the rotational part of the horizontal velocity field (e.g. Herring & Métais, 1989; Waite & Bartello, 2004) is studied. Idealized simulations in a cubic box of side $L = 2\pi$ are considered. The sharp spectral filter $k_c = \pi/\Delta$ is employed. Spatial derivatives are discretized using the spectral transform method with cubic truncation, along with the two-thirds rule for eliminating aliasing errors. The explicit third-order Adams-bashforth scheme is employed for time advancement.

Results and Discussion

The horizontal wavenumber energy spectra show an approximately $-5/3$ slope along with a bump around the buoyancy wavenumber k_b , when the filter scale Δ is small enough compared to the buoyancy scale L_b . This maximum limitation on Δ ensures that the dynamics of stratified turbulence, including Kelvin-Helmholtz instabilities, are captured by the LES. The criterion on LES of stratified turbulence depends on the SGS models: for the Kraichnan model $\Delta/L_b < 0.47$, for the Smagorinsky model $\Delta/L_b < 0.17$ and for the dynamic Smagorinsky model $\Delta/L_b < 0.24$. Our results show that the regular Smagorinsky model requires a filter scale that is respectively three times smaller than the Kraichnan model and almost two times smaller than the dynamic Smagorinsky model to reproduce similar results. In addition, the eddy dissipation spectra for stratified turbulence suggest an anisotropic energy transfer that happens at large horizontal and small vertical scales. Furthermore, the statistical analysis on the dynamic Smagorinsky coefficient c_s in LES of stratified turbulence demonstrates that large shears lead to small c_s and thereby reduce eddy dissipation (in line with results for unstratified turbulence, e.g. Wan & Porté-Agel, 2011). Overall, the dynamic Smagorinsky model is an approach that is applicable for LES of stratified turbulence in complex geometries and with a fair amount of computational costs.

References

- Germano M., Piomelli U., Moin P. & Cabot W. H. *Phys. Fluids A* **3**(7): 1760-1765, 1991.
- Herring J. R. & Métais O. *J. Fluid Mech.* **202**: 97-115, 1989.
- Kraichnan R. H. *J. Atmos. Sci.* **33**: 1521-1536, 1976.
- Smagorinsky J. *Mon. Weather Rev.* **91**(3): 99-164, 1963.
- Waite M. L. & Bartello P. J. *J. Fluid Mech.* **517**: 281-303, 2004.
- Wan F. & Porté-Agel F. *Boundary-Layer Meteorol* **138**: 367-384, 2011.

*corresponding author: sinakhani@uwaterloo.ca