Nonlinear evolution of the zigzag instability in stratified-rotating fluids

J. Basley[1]*, P. Billant[1]

[1]LadHyX, Ecole Polytechnique, UMR 7646 CNRS, 91128 PALAISEAU, FRANCE

While quasi-geostrophic turbulence is analogous to 2D turbulence with an inverse cascade of energy towards large scales, recent numerical simulations of stratified-rotating turbulence [1, 2] have revealed that an energy cascade towards small scales may also exist when the Rossby number is small but finite.

In order to highlight the mechanisms of generation of small scales in stratified-rotating fluids, we study the transition to turbulence of a columnar counter-rotating vortex pair by means of direct numerical simulations of the Navier-Stokes equations under the Boussinesq approximation.

Previous studies on stratified fluids without rotation [3, 4] have shown that the high vertical shear generated by the zigzag instability can indeed give rise to Kelvin-Helmholtz and gravitational instabilities, and eventually to a transition to small scale turbulence [5]. This occurs when the buoyancy Reynolds number ReF_h^2 is sufficiently large, where $F_h = \Gamma/(2\pi N R^2)$ is the horizontal Froude number and $Re = \Gamma/2\pi\nu$ is the Reynolds number, with Γ the circulation of the vortices, R the radius, N the Brunt-Väisälä frequency and ν the viscosity.

In presence of planetary rotation, we show that secondary instabilities can only develop when the Rossby number $Ro = \Gamma/(\pi f R^2)$ (with f the Coriolis parameter) is high enough, as seen in Figure 1. When the Rossby number is small ($Ro < 1$), the conservation of potential vorticity imposes a strong limit of the order Ro/F_h on the attainable vertical shear. Therefore the threshold for shear instabilities can be reached when Ro/F_h is higher than a critical value. This suggests that the shear instability pathway for the generation of small-scales can be active in stratified-rotating turbulence but only below a critical scale.

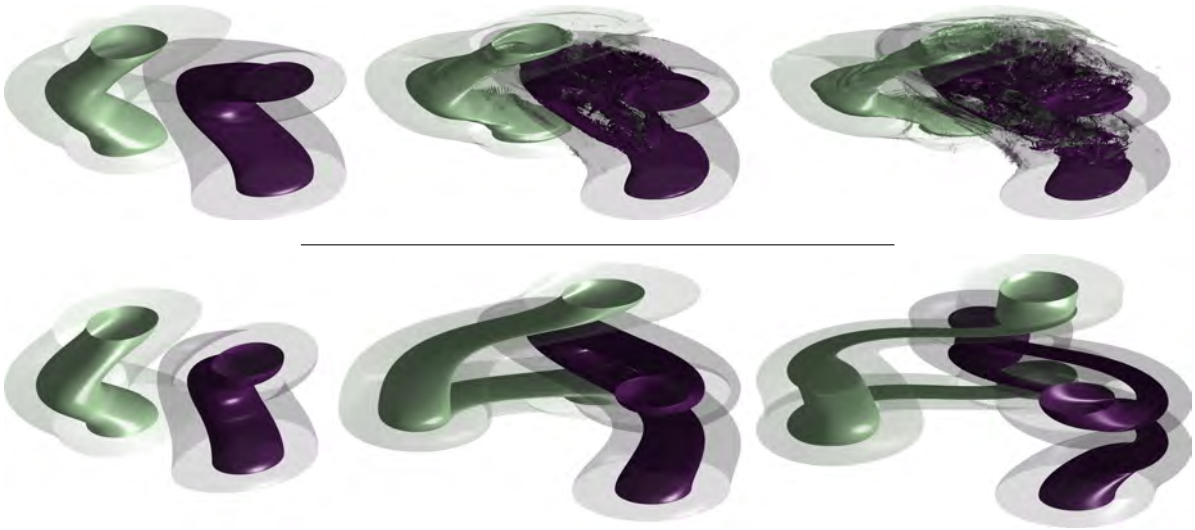

Fig. 1: Time evolution of a columnar vortex pair for two Rossby numbers: top, $Ro = 10$ and bottom, $Ro = 0.2$ for $Re = 5000$, $F_h = 0.4$. Two contours of potential vorticity are shown. A transition to turbulence is only observed for $Ro = 10$.

References

[1] Pouquet, A. & Marino, R., Geophysical turbulence and the duality of the energy flow across scales. *Phys. Rev. Lett.* **111** (23), 234501, 2013.
[2] Deusebio, E., Vallgren, A., Lindborg, E., The route to dissipation in strongly stratified and rotating flows. *J. Fluid Mech.* **720**, 66–103, 2013.
[3] Deloncle, A., Billant, P. & Chomaz, J.-M., Nonlinear evolution of the zigzag instability in stratified fluids: a shortcut on the route to dissipation. *J. Fluid Mech.* **599**, 229–239, 2008.
[4] Waite, M. L. & Smolarkiewicz, P. K., Instability and breakdown of a vertical vortex pair in a strongly stratified fluid, *J. Fluid Mech.* **606**, 239–273, 2008.
[5] Augier, P., Chomaz, J.-M., Billant, P., Spectral analysis of the transition to turbulence from a dipole in stratified fluid. *J. Fluid Mech.* **713**, 86–108, 2012.

*corresponding author: jeremy.basley@ladhyx.polytechnique.fr