Equipartition of divergent and rotational energy in stratified turbulence

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In numerical simulations of stratified turbulence [1] it has been observed that kinetic energy is often equipartitioned between rotational and divergent modes, respectively associated with the stream function, Ψ, and the velocity potential, Φ, in a Helmholtz decomposition of the horizontal velocity

$$\mathbf{u} = \mathbf{u}_r + \mathbf{u}_d = -\nabla \times (\mathbf{e}_z \Psi) + \nabla \phi \,, \tag{1}$$

where $\mathbf{e}_z$ is the vertical unit vector.

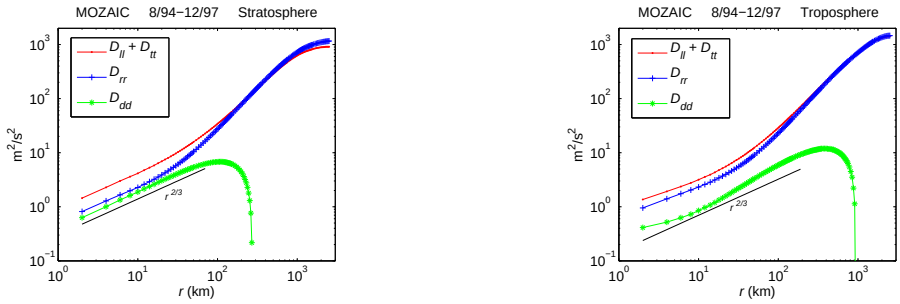

Fig. 1: Decomposition of $D_{ll} + D_{tt}$ into D_{rr} and D_{dd} in the lower stratosphere and the upper troposphere

It is not straightforward to make a Helmholtz decomposition from measurements. Longitudinal and transverse structure functions, $D_{ll} = \langle \delta u_l \delta u_l \rangle$ and $D_{tt} = \langle \delta u_t \delta u_t \rangle$, are often calculated from aircraft data. Here, δ denotes the increment between two points separated by a distance r, u_l and u_t the velocity components parallell and perpendicular to the aircraft track respectively and $\langle\rangle$ an average. In this contribution we show that the structure functions $D_{rr} = \langle \delta \boldsymbol{u}_r \cdot \delta \boldsymbol{u}_r \rangle$ and $D_{dd} = \langle \delta \boldsymbol{u}_d \cdot \delta \boldsymbol{u}_d \rangle$ can be calculated as

$$D_{rr} = D_{tt} + \int_0^r \frac{1}{r}(D_{tt} - D_{ll})\, \mathrm{d}r \,, \tag{2}$$

$$D_{dd} = D_{ll} - \int_0^r \frac{1}{r}(D_{tt} - D_{ll})\, \mathrm{d}r \,. \tag{3}$$

The decomposition is applied to structure functions calculated from aircraft data. In the lower stratosphere, D_{rr} and D_{dd} both show a nice $r^{2/3}$-dependence for $r \in [2,\, 20]$ km. In this range, the ratio between rotational and divergent energy is close to unity, as seen in the figure. In the upper troposphere, on the other hand, D_{rr} and D_{dd} show no clean $r^{2/3}$-dependence, although the overall slope of D_{dd} is close to $2/3$ for $r \in [2,\, 400]$ km. The ratio between rotational and divergent energy is around three for $r < 100$ km. We interpret these results as giving evidence of a clean case of stratified turbulence in the lower stratosphere, but an "unclean" case in the upper troposphere, where forcing due to release of latent heat is causing the deviation from the the expected results. We also argue that the results clearly point against the hypothesis recently expounded by [2] that inertia-gravity waves give rise to the observed structure functions or spectra.

References

[1] Lindborg, E. & Brethouwer, G. 2007 Stratified turbulence forced in rotational and divergent modes *J. Fluid Mech.*, **586**, 83-108

[2] Callies, J., Ferrari, R.& B ühler, O. 2014 Transition from geostrophic turbulence to inertia-gravity waves in the atmospheric energy spectrum *Proc. Nat. Acad. Sci.*, **111**, 17033-17038

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