Transient stability of stratified shear layers

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The stability and mixing of stratified shear layers is a canonical problem in fluid dynamics with relevance to flows in the ocean and atmosphere. The Miles-Howard theorem states that a necessary condition for normal-mode instability in parallel, inviscid, steady stratified shear flows is that the gradient Richardson number, Ri_g, is less than 1/4 somewhere in the flow. However, substantial transient growth of non-normal modes may be possible at finite times. We have calculated the 'optimal perturbations' associated with maximum perturbation energy gain for a hyperbolic-tangent shear layer with linear stable stratification (figure 1(a)) using a direct-adjoint looping method [1]. For the times considered here, the perturbations take the form of rolls tilted against the background shear and are able to grow via the Orr mechanism, similar to non-normal growth in unstratified flows [2]; however, some of the perturbation energy is found in wavelike structures outside of the shear layer for the more strongly stratified cases (see figure 1(b)). $O(10^1 - 10^2)$ gains in perturbation energy are found, even when $Ri_g > 1/4$ everywhere in the flow.

The linear optimal perturbations are then used to initialize direct numerical simulations. For small but finite initial perturbation amplitudes E_0, the optimal perturbations grow at the predicted linear rate initially, but then experience sufficient transient growth to become nonlinear and saturate. The flow is then susceptible to secondary instabilities which cause breakdown into turbulence and mixing of the background flow, even in flows for which $Ri_g > 1/4$ everywhere. We will describe the nonlinear evolution of the optimal perturbations and characterize the resulting turbulence and mixing for varying bulk Richardson number, Ri_b, and initial energy, E_0.

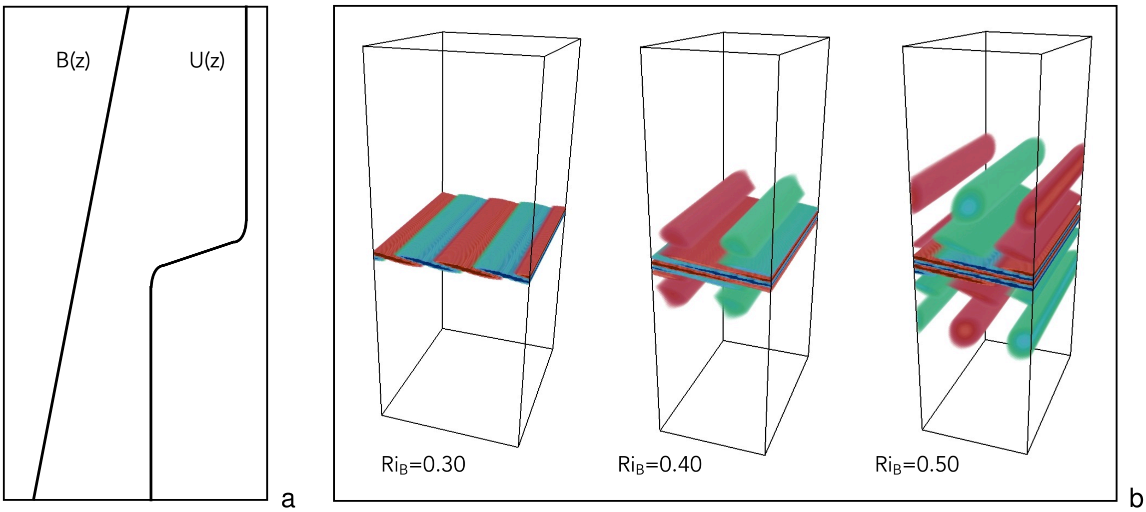

Fig. 1: (a) Background profiles of buoyancy, $B(z)$, and velocity, $U(z)$. (b) Vertical velocity of linear optimal perturbations for a nondimensional target time $T = 15$ and bulk Richardson numbers $Ri_B = 0.30, 0.40, 0.50$. The red and blue/green colours denote negative and positive velocities, respectively.

References

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[2] Arratia C, Caulfield CP & Chomaz J-M. *J Fluid Mech* **717**:90-133, 2013.

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